MAGNTS 2022 Poster Session

Sushmanth Jacob Akkarapakam, Indiana University–Purdue University Indianapolis

Title: Periodic points of an algebraic function related to a Ramanujan's continued fraction

Abstract: A continued fraction $v(\tau)$ of Ramanujan is evaluated at certain arguments in the field $K = \mathbb{Q}(\sqrt{-d})$, with $-d \equiv 1 bmod 8$, in which the ideal $(2) = \mathfrak{p}_2\mathfrak{p}_2'$ is a product of two prime ideals. These values of $v(\tau)$ are shown to generate the inertia field of $\mathfrak{p}_2$ or $\mathfrak{p}_2'$ in an extended ring class field over the field K. The same values are also shown to be periodic points of a fixed algebraic function $F*(x)$, independent of d. These are analogues of similar results for the Rogers-Ramanujan continued fraction.

Rusiru Gambheera, University of California, San Diego

Title: Fitting Ideals of (S,T)-modified Iwasawa Modules over CM Fields

Abstract: Recently, Bley and Popescu proved an equivariant main conjecture along any rank one, sign-normalized Drinfeld modular (geometric) Iwasawa–tower of a general function field of characteristic p. Their module of interest resonates very much with the Ritter-Weiss module used by Dasgupta and Kakde in their proof of Brumer-Stark conjecture away from 2. Recently, we proved a generalization of the main theorem in the Dasgupta-Kakde paper in a certain special case. Then, we used this result to prove a new unconditional equivariant main conjecture for a Ritter-Weiss-type module at the top of the cyclotomic Iwasawa tower over a CM number field, by computing the Fitting ideal of these new modules over the appropriate profinite group ring. This is a number field analogue of the Bley-Popescu result. As an application we compute the Fitting ideal of a certain naturally arising Iwasawa module. This is joint work with Cristian Popescu.

Oussama Hamza, Western University

Title: Filtrations, Mild groups and Arithmetic in an Isotypical context

Abstract: Pro-p groups arise naturally in number theory as quotients of absolute Galois groups over number fields. These groups are quite mysterious. During the 60's, Koch gave a presentation of some of these quotients. Furthermore, around the same period, Jennings, Golod, Shafarevich and Lazard introduced two integer sequences (a_n) and (c_n), closely related to a special filtration of a finitely generated pro-p group G, called the Zassenhaus filtration. These sequences give the cardinality of G, and characterize its topology. For instance, we have the well-known Gocha's alternative (Golod and Shafarevich): There exists an integer n such that $a_n = 0$ (or c_n has an exponential growth) if and only if G is a Lie group over p-adic fields.

In 2016, Minac, Rogelstad and Tan inferred an explicit relation between a_n and c_n. Recently (2022), considering geometrical ideas of Filip and Stix, Hamza got more precise relations in an isotypical context: when the automorphism group of G admits a subgroup of order a prime q dividing $p-1$.

During the last decade, using Anick's results, Labute and Minac introduced Mild groups, which have cohomological dimension 2. Thanks to Koch's presentation, Labute was able to

construct finitely presented mild quotients of the absolute Galois group of $\mathbb{Q}$. In 2020, Maire and Hamza construct mild quotients of the absolute Galois group of a number fields, defined by an infinite number of relations.

This poster will mostly review some results of Golod, Shafarevic, Koch, Lazard, Anick, Labute, Minac, Tân, Forré, Maire and Hamza. We also give several examples.

Qiao He, University of Wisconsin, Madison
Title: A proof of the Kudla-Rapoport conjecture for Kramer models
Abstract: Abstract: We prove the Kudla-Rapoport conjecture for Krämer models of unitary Rapoport-Zink spaces at ramified places. It is a precise identity between arithmetic intersection numbers of special cycles on Kramer models and modified derived local densities of hermitian forms. As an application, we relax the local assumptions at ramified places in the arithmetic Siegel-Weil formula for unitary Shimura varieties, which is in particular applicable to unitary Shimura varieties associated to unimodular hermitian lattices over imaginary quadratic fields. This is a joint work with Chao Li, Yousheng Shi and Tonghai Yang.

Grace Jaffe, University of Illinois, Urbana-Champaign
Title: A Frobenius Perspective on the Stern Sequence
Abstract: The Frobenius problem asks, given a finite set of relatively prime positive integers, what is the maximum integer not expressible as a linear combination of the set elements using only non-negative integer coefficients? The Stern sequence is defined by $s(0) = 0, s(1) = 1$, and recursively thereafter by $s(2n) = s(n) and s(2n+1) = s(n) + s(n+1)$. The interaction of these two objects forms a variation on each in which we examine linear combinations of two elements where the coefficients themselves are also relatively prime. We show there always exists some maximum integer that cannot be expressed in this form and present further results.

John Sung Min Lee, University of Illinois, Chicago
Title: On the Acyclicity of Reductions of Elliptic Curves for Primes in Arithmetic Progressions
Abstract: Let E be an elliptic curve defined over $\mathbb{Q}$. Let us denote the reduction of E modulo a prime p by $\tilde{E}_p$. Recently, assuming Generalized Riemann Hypothesis, Akbal and Güloğlu calculated the density of primes p in an arithmetic progression for which $\tilde{E}_p(\mathbb{F}_p)$ is cyclic, and raised a question when this density vanishes. I am going to share my recent result on this question; the poster includes expressing the density as an almost Euler product, and generating a family of elliptic curves for which $\tilde{E}_p(\mathbb{F}_p)$ is NOT cyclic for all but finitely many primes, given $p \equiv k \pmod{n}$. This is a joint work with Nathan Jones.

Yeqin Liu, University of Illinois, Chicago
Title: The Cohomology of Spherical Vector Bundles on $K3$ Surfaces
Abstract: On a polarized $K3$ surface (X, H) over $\mathbb{C}$, a vector bundle E is called spherical if it is H-stable and rigid ($\mathrm{ext}^1(E, E) = 0$). In this poster, I introduce an algorithm to compute their exact numbers of global sections by using Bridgeland stability. It is very likely that the

method generalizes to compute the cohomology of a general member of any moduli space of stable sheaves on a $K3$ surface.

Lucas Mioranci, University of Illinois, Chicago
Title: Normal Bundles of Rational Normal Curves on Hypersurfaces
Abstract: Abstract: Let C be the rational normal curve of degree e in $\P^n$, and let $X \subset \P^n$ be a degree $d \geq 2$ hypersurface containing C. I. Coskun and E. Riedl showed that the normal bundle $N_{C/X}$ is balanced for a general X. H. Larson studied the case of lines ($e = 1$) and computed the dimension of the space of hypersurfaces for which $N_{C/X}$ has a given splitting type. This poster is based on a paper where we work with any $e \geq 2$. We compute explicit examples of hypersurfaces for all possible splitting types, and for $d \geq 3$, we compute the dimension of the space of hypersurfaces for which $N_{C/X}$ has a given splitting type. For $d = 2$, we give a lower bound on the maximum rank of quadrics with fixed splitting type.

Evan O'Dorney, University of Notre Dame
Title: Reflection theorems for number rings
Abstract: In 1997, Y. Ohno discovered (quite by accident) a beautiful reflection identity relating the number of cubic rings, equivalently binary cubic forms, of discriminants D and $-27D$. In the case that is squarefree, this corresponds to Scholz's 1932 reflection principle comparing the 3-class groups of the quadratic fields of discriminants D and $-3D$. Ohno's conjectured identity was proved in 1998 by Nakagawa. In this work, I present a new and more general method for proving reflection identities of this type, based on Poisson summation on adelic cohomology (in the style of Tate's thesis). Results from this method include reflection theorems for cubic forms over a general global field, as well as quadratic forms and quartic forms and rings. In particular, I obtain an equidistribution identity for triple covers of a curve over a finite field.

Sun Woo Park, University of Wisconsin, Madison
Title: On the prime Selmer ranks of cyclic prime twist families of elliptic curves over global function fields
Abstract: Fix a prime number p. Let $\mathbb{F}_q$ be a finite field of characteristic coprime to 2, 3, and p, which also contains the primitive p-th root of unity μ_p. Let E be a non-isotrivial elliptic curve over the global function field $K = \mathbb{F}_q(t)$ with minimal Weierstrass model given by $E : y^2 = x^3 + Ax + B$. Denote by $F_n(\mathbb{F}_q)$ the set of polynomials of degree n over the finite field $\mathbb{F}_q$. Suppose that $\mathrm{Gal}(K(E[p])/K) = SL_2(\mathbb{F}_p) \rtimes T$ for some finite cyclic subgroup T. Let $\chi \in \mathrm{Hom}(\mathrm{Gal}(\overline{K}/K), \mu_p)$ be a cyclic order p character, with $L^\chi/K := K(\sqrt[p]{f})/K$ the associated cyclic p-extension of K for some p-th power free polynomial $f \in F_n(\mathbb{F}_q)$. Denote by E^χ the kernel of the norm map $N : \mathrm{Res}_K^{L^\chi} E \to E$, where $\mathrm{Res}_K^L E$ denotes the Weil restriction of scalars of E/K with respect to the field extension L/K. Let π be the unique prime ideal of the group ring $\mathbb{Z}[\mathrm{Gal}(L^\chi/K)] \cong \mathbb{Z}[\zeta_p]$ lying above the prime ideal $(p) \subset \mathbb{Z}$. Fix a positive constant $0 < \rho < 1$. Based on the works by Swinnerton-Dyer and Klagsbrun, Mazur, and Rubin, we prove that the probability distribution of the size of π-Selmer groups over the family of non-isotrivial elliptic curves E^χ/K, with a place of split multiplicative reduction, that are twisted by cyclic order-p characters $\chi \in \mathrm{Hom}(\mathrm{Gal}(\overline{K}/K), \mu_p)$ associated to the cyclic p-extension $L^\chi/K := K(\sqrt[p]{f})/K$ for

some polynomial $f \in F_n(\mathbb{F}_q)$ converges to the distribution conjectured by Bhargava, Kane, Lenstra, Poonen, and Rains for sufficiently large degree n, with error bounds of order $n^{-\alpha}$ for some positive $0 < \alpha < \min\left(\rho \ln \rho + 1 - \rho,\ -\rho \ln\left(1 - \frac{1}{|T|}\frac{p}{p^2-1}\right)\right)$. For quadratic twist families of elliptic curves, the maximal attainable error bounds are given by approximately $n^{-0.3339}$. The key tools used in proving these results are the Riemann hypothesis over global function fields, the Erdös-Kac theorem, and the geometric ergodicity of Markov chains.

Tian Wang, University of Illinois, Chicago
Title: Effective Serre's open image theorem for a product of elliptic curves
Abstract: Let $E_1/\mathbb{Q}, \ldots, E_n/\mathbb{Q}$ be pairwise non-isogenous elliptic curves without complex multiplication. For each $1 \leq i \leq n$ denote the conductor of each E_i by N_{E_i}. For each prime ℓ, we consider the the ℓ-adic Galois representation $\rho_{A,\ell}$ of the product $A = \prod_{1 \leq i \leq n} E_i$. Then, assuming GRH, we give an effective bound for the size of the largest prime ℓ for which $\rho_{A,\ell}$ is not surjective via N_{E_i}. This is a joint work in progress with J. Mayle and is a generalization of the result by J. Myle and T. Wang for a non-CM elliptic curve in 2021.

Zhining Wei, The Ohio State University
Title: The strong multiplicity one for GL_n and its applications
Abstract: In this work, I will establish a new type strong multiplicity one theorem for GL_n. As an application, we can obtain some multiplicity one results for the paramodular forms. This is a joint work with Pan Yan and Shaoyun Yi.

Wern Yeong, University of Notre Dame
Title: The algebraic Green-Griffiths-Lang conjecture for the complement of a generic quartic plane curve
Abstract: The Frobenius problem asks, given a finite set of relatively prime positive integers, what is the maximum integer not expressible as a linear combination of the set elements using only non-negative integer coefficients? The Stern sequence is defined by $s(0) = 0, s(1) = 1$, and recursively thereafter by $s(2n) = s(n) and s(2n+1) = s(n) + s(n+1)$. The interaction of these two objects forms a variation on each in which we examine linear combinations of two elements where the coefficients themselves are also relatively prime. We show there always exists some maximum integer that cannot be expressed in this form and present further results.

John Yin, University of Wisconsin, Madison
Title: Counting Rational Points on $\mathrm{Sym}^2 \mathbb{P}^1$
Abstract: The Batyrev-Manin conjecture predicts point counts on Fano varieties with respect to the anti-canonical height. We prove the analog of this conjecture for stacks, as formulated by Ellenberg-Satriano-Zureick Brown, in the case of $\mathrm{Sym}^2 \mathbb{P}^1$.