2023 MAGNTS POSTER SESSION

SATURDAY, OCTOBER 7, 2023

Raghavendra N Bhat (UIUC):

On Square-Prime numbers and filtered rays over iterated absolute differences on layers of integers

The dynamical system generated by the iterated calculation of the high order gaps between neighboring terms of a sequence of natural numbers is remarkable and only incidentally characterized at the boundary by the notable Proth-Glibreath Conjecture for prime numbers. We introduce a natural extension of the original triangular arrangement, obtaining a growing hexagonal covering of the plane. We focus on the sequence of Square-Primes (products of squares and primes) and derive some preliminary results on these numbers. We also prove results on their distribution and provide some conjectures. This is joint work with Alexandru Zaharescu and Cristian Cobeli.

Sridhar Venkatesh (University of Michigan):

Higher derived pushforwards of log differentials on toric varieties

Let $f : \tilde{X} \to X$ be a resolution of singularities such that f is an isomorphism over the smooth locus of X and $E := f^{-1}(X_{\text{sing}})_{\text{red}}$ is a simple normal crossing divisor. Inspired by the notion of higher rational singularities, introduced by Friedman and Laza, we investigate the vanishing (and non-vanishing) of the higher derived pushforwards $R^i f_* \Omega^p_{\tilde{X}}(\log E)$, in the case where X is a toric variety. This is part of joint work with Anh Duc Vo and Wanchun Shen.

Shifan Zhao (OSU):

On Mobius functions from automorphic forms and a generalized Sarnak's conjecture.

We consider Mobius functions associated with two types of L-functions: Rankin-Selberg L-functions of symmetric powers of distinct holomorphic cusp forms and L-functions derived from Maass cusp forms. We show that these Mobius functions are weakly orthogonal to bounded sequences. As a direct corollary, a generalized Sarnak's conjecture holds for these two types of Mobius functions.

Shitan Xu (MSU):

Stably rationality of Brauer-Severi surface bundles over rational 3-folds.

We prove a sufficient condition for a Brauer-Severi surface bundle over a rational 3-fold to be not stably rational. We provide an example that satisfies this condition and show existence of families of Brauer-Severi surface bundles whose very general member is smooth and not stably rational.

Ali Alsetri (UKY):

Upper bound on dimension of Hilbert cubes contained in the Quadratic residues of $\mathbb{F}_p$.

We consider the problem of bounding the dimension of Hilbert cubes in a finite field $\mathbb{F}_p$ that do not contain any primitive roots. We show that the dimension of such Hilbert cubes is $O_\epsilon(p^{\frac{1}{8}+\epsilon})$ for any $\epsilon > 0$, matching what can be deduced from the classical Burgess estimate in the special case when the Hilbert cube is an arithmetic progression. This is joint work with Xuancheng Shao.

Peikai Qi (MSU):

Iwasawa lambda invariant and Massey products

How does the class group of the number field change in field extensions? This question is too wild to have a uniform answer, but there are some situations where partial answers are known. I will compare two such situations. First, in Iwasawa theory, instead of considering a single field extension, one considers a tower of fields and estimates the size of the class groups in the tower in terms of some invariants called λ and μ. Second, in a paper of Lam-Liu-Sharifi-Wake-Wang, they relate the relative size of Iwasawa modules to values of a "generalized Bockstein map", and further relate these values to Massey products in Galois cohomology in some situations. I will compare these two approaches to give a description of the cyclotomic Iwasawa λ-invariant of some imaginary quadratic fields and other fields in terms of Massey products.

Yu Shen (MSU):

Derived categories of maximal orders.

We consider a quaternion algebra A over $\mathbb{P}^2$ that is ramified on a smooth quartic curve. Such an algebra is an example of an exotic del Pezzo order. We consider the moduli functor of simple A-modules (with appropriate boundedness conditions) for a torsion-free simple algebra A. The coarse moduli space always exists for such a functor. We construct a Fouerier-Mukai functor from the derived category of this moduli space into the derived category of A-modules. This is a new direction in studying derived categories of maximal orders.

Sushmanth Jacob Akkarapakam (Mizzou):

On the periodic points of the Ramanujan's cubic continued fraction

In this paper, we study the Ramanujan's cubic continued fraction $c(\tau)$ and describe how the periodic points for prescribed function arise as values of this continued fraction. We consider discriminants of the form $-d \equiv 1 \pmod 3$ and arguments in the field $K = \mathbb{Q}(\sqrt{-d})$. We let R_K be the ring of integers in this field and the prime ideal factorization of (3) in R_K be $(3) = \wp_3 \wp_3'$. We denote by Ω_f the ring class field over K whose conductor is f corresponding to the order R_{-d} of discriminant $-d = d_K f^2$ in K (d_K is the discriminant of K). We show that certain values of $c(\tau)$, for τ in the imaginary quadratic field $K = \mathbb{Q}(\sqrt{-d})$ with discriminant $-d \equiv 1 \pmod 3$, are periodic points of a fixed algebraic function, independent of d, and generate certain class fields Ω_{2f} over K.

Evan M. O'Dorney (CMU):

Reflection theorems for number rings

In 1997, Y. Ohno discovered (quite by accident) a beautiful reflection identity relating the number of cubic rings, equivalently binary cubic forms, of discriminants D and $-27D$. In the case that D is squarefree, this corresponds to Scholz's 1932 reflection principle comparing the 3-class groups of the quadratic fields of discriminants D and $-3D$. Ohno's conjectured identity was proved in 1998 by Nakagawa. In this work, I present a new and more general method for proving reflection identities of this type, based on Poisson summation on adelic cohomology (in the style of Tate's thesis). Results from this method include reflection theorems for cubic forms over a general global field, as well as quadratic forms and quartic forms and rings. In particular, I obtain an equidistribution identity for triple covers of a curve over a finite field.

Nick Geis (OSU):

Sign counting function for random multiplicative functions

Let f be a Rademacher random multiplicative function and let $M_f(x) := \sum_{n \leq x} f(n)$. Let $V_f(x)$ denote the number of sign changes of $M_f(x)$ up to x. We show that for any $c > 2$ that $V_f(x) = \Omega((\log\log\log x)^{1/c})$ as $x \to \infty$, almost surely.

John Yin (UW Madison):

Chebotarev Density Theorem over Local Fields

We compute the p-adic densities of points with a given splitting type along a finite map, analogous to the classical Chebotarev theorem over number fields and function fields. Under certain niceness hypotheses, we prove that these densities satisfy a functional equation in the size of the residue field. As a consequence, we prove a conjecture of Bhargava, Cremona, Fisher, and Gajović on factorization densities of p-adic polynomials.

The key tool is the notion of *admissible pairs* associated to a group, which we use as an invariant of the inertia and decomposition action of a local field on the fibers of the finite map. We compute the splitting densities by Möbius inverting certain p-adic integrals along the poset of admissible pairs. The conjecture on factorization densities follows immediately for tamely ramified primes from our general results. We reduce the complete conjecture (including the wild primes) to the existence of an explicit "Tate-type" resolution of the "resultant locus" over Spec $\mathbb{Z}$ and complete the proof of the conjecture by constructing this resolution.

Kiseok Yeon (Purdue):

The local solubility for homogeneous polynomials with random coefficients over thin sets

Let d and n be natural numbers greater or equal to 2. Let $\langle \boldsymbol{a}, \nu_{d,n}(\boldsymbol{x})\rangle \in \mathbb{Z}[\boldsymbol{x}]$ be a homogeneous polynomial in n variables of degree d with integer coefficients $\boldsymbol{a}$, where $\langle \cdot, \cdot \rangle$ denotes the inner product, and $\nu_{d,n} : \mathbb{R}^n \to \mathbb{R}^N$ denotes the Veronese embedding with $N = \binom{n+d-1}{d}$. Consider a variety $V_{\boldsymbol{a}}$ in $\mathbb{P}^{n-1}$, defined by $\langle \boldsymbol{a}, \nu_{d,n}(\boldsymbol{x})\rangle = 0$. In this paper, we examine a set of these varieties defined by

$$\mathbb{V}^P_{d,n}(A) = \{V_{\boldsymbol{a}} \subset \mathbb{P}^{n-1} | \ P(\boldsymbol{a}) = 0, \ \|\boldsymbol{a}\|_\infty \leq A\},$$

where $P \in \mathbb{Z}[\boldsymbol{x}]$ is a non-singular form in N variables of degree k with $2 \leq k \leq C(n,d)$ for some constant $C(n,d)$ depending at most on n and d. Suppose that $P(\boldsymbol{a}) = 0$ has a nontrivial integer solution. We confirm that the proportion of varieties $V_{\boldsymbol{a}}$ in $\mathbb{V}^P_{d,n}(A)$, which are everywhere locally soluble, converges to a constant c_P as $A \to \infty$. In particular, if there exists $\boldsymbol{b} \in \mathbb{Z}^N$ such that $P(\boldsymbol{b}) = 0$ and the variety $V_{\boldsymbol{b}}$ in $\mathbb{P}^{n-1}$ admits a smooth $\mathbb{Q}$-rational point, the constant c_P is positive.

Yizhen Zhao (MSU):

Period-Index Problems over Characteristic p

The period-index problem is a longstanding question about the Brauer groups of a field. This problem concerns the order (*the period*) of an element of the Brauer group and the index which is the gcd of all the degrees of finite splitting fields. I will focus on the p-torsion part of Brauer groups over characteristic p and the connection between the Kato's Swan conductor and period-index problems of discretely valued fields. As an application, I will talk about our progress on the period-index problem for the Brauer group of the function field of an algebraic curve over $\bar{k}((t))$, where k is a field of characteristic $p > 0$.

Jiaqi Hou (UW Madison):

Bounds for periods of eigenfunctions on arithmetic hyperbolic 3-manifolds

Let X be a compact arithmetic hyperbolic 3-manifold and Y a hyperbolic surface in X. Let ψ be a Hecke-Maass form on X, which is a joint eigenfunction of the Laplacian and Hecke operators. We consider the period integral of ψ along Y as the spectral parameter tends to infinity. Zelditch proved the general local bounds for the periods of Laplace eigenfunctions. We prove a power saving bound for the period of ψ along Y over the local bound. The result is based on the method of arithmetic amplification.

Zhining Wei (Brown):

The strong multiplicity one for paramodular forms and its applications

In this work, I will introduce the spinor L-functions and standard L-functions for paramodular forms for GSp_4. I will prove some strong multiplicity one result for them. As an application, I will show that paramodular forms can be determined by its twisted L-values. This is a joint work with Xiyuan Wang, Pan Yan and Shaoyun Yi.

Som Sadashiv Phene (UM):

Polarization Invariance of Geometric Quantization

We study real vs complex polarization of semi toric manifolds such as K3 surface under Geometric Quantization. One construction of the fibration is given by Toric degeneration. Having both Symplectic and holomorphic structure, we can compare the Hilbert space obtained from Lagrangian fibration with singularity to the case of Kahler polarization. This finds application to Mirror Symmetry under large complex structure limit.

Auden Hinz (UIC):

Upper bounds related to an isogeny criterion for elliptic curves

For E_1 and E_2 elliptic curves defined over a number field K, without complex multiplication, we consider the function $\mathcal{F}_{E_1,E_2}(x)$ counting non-zero prime ideals $\mathfrak{p}$ of the ring of integers of K, of good reduction for E_1 and E_2, of norm at most x, and for which the Frobenius fields $\mathbb{Q}(\pi_{\mathfrak{p}}(E_1))$ and $\mathbb{Q}(\pi_{\mathfrak{p}}(E_2))$ coincide. Motivated by an isogeny criterion of Kulkarni, Patankar, and Rajan, which relates isogeny to the growth of $\mathcal{F}_{E_1,E_2}(x)$, we prove upper bounds for $\mathcal{F}_{E_1,E_2}(x)$ in the non-isogenous case.

William C. Newman (OSU):

Injective Quadratic Self-Maps on $\mathbb{P}^2$

In this poster, I describe a criterion for when a quadratic rational map $\mathbb{P}^2 \to \mathbb{P}^2$ defined over a field K is injective on K-rational points, along the way giving some partial results for $\mathbb{P}^n$. When K is a finite field, the criterion is very restrictive, and we are able to explicitly describe exactly which self-maps of $\mathbb{P}^2$ are injective (hence, bijective) on K rational points. This is joint work with Michael Zieve.

Matthew Hase-Liu (Columbia):

The mapping space of a smooth projective curve to a smooth hypersurface of low degree

Browning and Vishe analyzed the space of rational curves on smooth hypersurfaces of low degree by a clever use of spreading out and an application of the Hardy-Littlewood circle method. We reinterpret the circle method geometrically, allowing us to prove a generalization for a fixed smooth projective curve.

Yeqin Liu (UIC):

Spherical vector bundles on nodal K3 surfaces

Stable sheaves on K3 surfaces and their moduli spaces have been well studied, however less is understood when the underlying surface acquires singularities. We start approaching this problem by studying stable spherical bundles on nodal K3 surfaces. When the Picard rank is 1, we classify all such bundles and see that, the existence of stable sheaves of certain Chern classes can be obstructed when the underlying surface becomes singular.

Laurence Petrus Wijaya (UKY):

On nonzero coefficients of binary cyclotomic polynomials

Let $\vartheta(m)$ is number of nonzero coefficients in the m-th cyclotomic polynomial. For real $\gamma > 0$ and $x \geq 2$ we define

$$H_\gamma(x) = \#\left\{m : \ m = pq \leq x, \ p < q \text{ primes }, \ \vartheta(m) \leq m^{1/2+\gamma}\right\},$$

and show that for any fixed $\eta > 0$, uniformly over γ with

$$9/20 + \eta \leq \gamma \leq 1/2 - \eta,$$

we have an asymptotic formula

$$H_\gamma(x) \sim C(\gamma)x^{1/2+\gamma}/\log x, \qquad x \to \infty,$$

where $C(\gamma) > 0$ is an explicit constant depending only on γ. This extends the previous result of É. Fouvry (2013), which has $12/25$ instead of $9/20$.

Lucas Mioranci (UIC):

Algebraic hyperbolicity of very general hypersurfaces in homogeneous varieties

A complex projective variety X is *(Brody) hyperbolic* when it admits no nonconstant holomorphic map $\mathbb{C} \to X$, that is, when it contains no entire curves. In dimension one, hyperbolic curves are those with genus greater or equal to 2. In higher dimensions, it is a difficult and important problem to characterize hyperbolic varieties. It motivates celebrated conjectures such as the Lang Conjectures, Green-Griffiths Conjecture, and Manin's Conjecture.

Algebraic hyperbolicity has been introduced as an algebraic analogue for hyperbolicity: we say X is algebraically hyperbolic if there exists an ample divisor H and a real number $\epsilon > 0$ such that the geometric genus $g(C)$ and the degree of any integral curve $C \subset X$ satisfy the inequality

$$2g(C) - 2 \geq \epsilon \deg_H(C).$$

In particular, algebraically hyperbolic varieties do not contain any rational or elliptic curves. Every hyperbolic variety is algebraically hyperbolic, and Demailly conjectured that the converse holds.

The algebraic hyperbolicity of very general hypersurfaces in projective space is almost completely classified by the results of Clemens, Ein, Voisin, Pacienza, Coskun and Riedl, and Yeong. By building on their techniques, I extended the classification to the much more general case of homogeneous varieties, thus obtaining explicit bounds for the hyperbolicity in plenty of open cases, including Grassmannians, flag varieties, and their products.

Cruz Castillo (UIUC):

Sign changes of the error term in the Piltz divisor problem

For an integer $k \geq 3$, let $\Delta_k(x) := \sum_{n\leq x} d_k(n) - \text{Res}_{s=1}(\zeta^k(s)x^s/s)$, where $d_k(n)$ is the k-fold divisor function, and $\zeta(s)$ is the Riemann zeta-function. In the 1950's, Tong showed for all large enough X, $\Delta_k(x)$ changes sign at least once in the interval $[X, X + C_k X^{1-1/k}]$ for some positive constant C_k. For a large parameter X, we show that if the Lindelöf hypothesis is true, then there exist many disjoint subintervals of $[X, 2X]$, each of length $X^{1-\frac{1}{k}-\varepsilon}$, such that $\Delta_k(x)$ does not change sign in any of these subintervals. If the Riemann hypothesis is true, then we can improve the length of the subintervals to $\gg X^{1-\frac{1}{k}}(\log X)^{-k^2-2}$. These results may be viewed as higher-degree analogues of a theorem of Heath-Brown and Tsang, who studied the case $k = 2$. This is joint work with Siegfried Baluyot.