

Random walks in number theory

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Abstract: We will stroll through various results in number theory that bear connections with the theory of random walks. There is a surprising number of such links, beginning with Billingsley’s classical result that the sequence of prime factors, appropriately normalized, converges to a Brownian motion. More recently, Ford used a “ballot theorem” to obtain sharp estimates on the distribution of integers with a divisor in a given interval. Ballot theorems also appear in the theory of Branching Random Walks and, more generally, of log-correlated fields. These probabilistic objects have been featured in recent striking work by various authors on the distribution of the maximum of the Riemann zeta function in a randomly selected interval, as well as in the theory of random multiplicative functions. And they also emerge when studying the distribution of the complex divisor function.