Arithmetic Dynamics, Arithmetic Geometry, and Number Theory

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The core of *arithmetic geometry* is the study of points on a variety X defined over a non-algebraically closed field K such as $\mathbb{Q}$, $\mathbb{Q}_p$, or $\mathbb{F}_p$. The core of *discrete dynamical systems* is the study of orbits of points under iteration of a function $f : X \to X$. Toss them into a pot and stir vigorously, out comes the new field of ***arithmetic dynamics***, in which one studies arithmetic properties of the orbits of points in $X(K)$. Many familiar dishes from the arithmetic geometry of abelian varieties have natural analogs in aritmetic dynamics, including for example uniform boundedness of torsion (Mazur–Merel), rational points on subvarieties (Faltings, née Mordell–Lang conjecture), torsion points on subvarieites (Raynaud, née Manin–Mumford conjecture), and special points on moduli spaces (André–Oort conjecture). In my first talk I will explain these parallels in detail, describe conjectures and partial results on the dynamical side, and mention some parts of arithmetic geometry, such as the theory of Hecke operators, that do not (yet) have dynamical analogs. In my second talk I will delve more deeply into one or two topics from arithmetic dynamics.