2025 MAGNTS POSTER SESSION

SATURDAY, MARCH 29, 2025

Ali Alsetri (University of Kentucky):
Density versions of the binary Goldbach problem

Let $\delta > 1/2$. We prove that if A is a subset of the primes such that the relative density of A in every reduced residue class is at least δ, then almost all even integers can be written as the sum of two primes in A. The constant $1/2$ in the statement is best possible. Moreover we give an example to show that for any $\epsilon > 0$ there exists a subset of the primes with relative density at least $1 - \epsilon$ such that $A + A$ misses a positive proportion of even integers. This is joint work with Xuancheng Shao.

Cruz Castillo and Debmalya Basak (UIUC):
Surfaces Associated to Zeros of Automorphic L-functions

Assuming the Riemann Hypothesis, Montgomery established results concerning pair correlation of zeros of the Riemann zeta function. Rudnick and Sarnak extended these results to automorphic L-functions and all level correlations. We show that automorphic L-functions exhibit additional geometric structures related to the correlation of their zeros. This is joint work with Alexandru Zaharescu.

Yen-Tsung Chen (Penn State University):
Analytic continuation of Kochubei multiple polylogarithms and its applications

We propose an analytic continuation of Kochubei multiple polylogarithms using the techniques developed by Furusho in 2022. They satisfy the same system of difference equations as the original ones. Moreover, we produce a family of linear relations and a linear independence result for values of our analytically continued Kochubei polylogarithms at algebraic elements from a cohomological aspect

Pavel Coupek (Michigan State University):
Ramification bounds via Wach modules and q-crystalline cohomology

Let K be an absolutely unramified p-adic field. We describe a bound on ramification of a mod p G_K-representation associated with a Wach module of height at most i. Using q-crystalline cohomology, this provides a new ramification bound for the i-th mod p etale cohomology for a smooth and proper p-adic formal scheme over $\mathcal{O}_K$ (for arbitrary i).

Andy Day (Penn State):
Algebraic Skew Embedding for Curves

Given a smooth manifold X, a totally skew embedding of X is an embedding of X into an euclidean space $\mathbb{R}^N$ such that for any two distinct points $x, y \in X$, their embedded tangent spaces in $\mathbb{R}^N$ neither intersect nor contain parallel lines. The concept can be generalized to algebraic skew embeddings of complex smooth varieties into complex projective spaces. In this work, we establish an upper bound and a lower bound of the minimal dimension N such that there exists a skew embedding into the space $\mathbb{C}P^N$ for a given smooth variety X. In particular, we classify the algebraic curves in terms of their minimal skew embedding dimension N.

Juan Pablo De Rasis (OSU):

Efficient computation of Fourier Coefficients of ETA-quotients

The partition function p(n) counts the number of ways to express n as the sum of positive integers, up to order. Different variations of this function such as overpartitions, partitions with parts relatively prime to two given prime numbers, partitions into odd parts, etc. have also been studied. Here we offer a computational analysis of the Fourier Coefficients of a general collection of eta-quotients, by showing that different parameters for these eta-quotients give rise to all these variations of partition functions, thus providing a general computation method for them; hence generalizing the analogous results of Lehmer (1938) which give an efficient way to compute the original partition function in terms of closed formulas for Kloosterman Sums. This is a joint work with Adrián Barquero-Sánchez, Nicolás Sirolli, and Jean Carlos Villegas-Morales.

Hunter Handley (OSU):

First-order definability of Darmon points in number fields

For a given number field K, we give a $\forall\exists\forall$-first order description of affine Darmon points over $\mathbb{P}^1_K$, and show that this can be improved to a $\forall\exists$-definition in a remarkable particular case. Darmon points, which are a geometric generalization of perfect powers, constitute a non-linear set-theoretical filtration between K and its ring of S-integers, the latter of which can be defined with universal formulas, as has been progressively proven by Koenigsmann, Park, and Eisenträger & Morrison. We also show that our formulas are uniform with respect to all possible S, with a parameter-free uniformity, and we compute the number of quantifiers and a bound for the degree of the defining polynomial. This is joint work with Juan Pablo De Rasis.

Tinghao Huang (OSU):

Simultaneous non-vanishing of central values of twisted Maass L-functions

In this paper, we establish a new quantitative result for the simultaneous non-vanishing of the central values of quadratically twisted L-functions associated with Hecke-Maass cusp forms. For a fixed Hecke-Maass cusp form Φ for $SL(2,\mathbb{Z})$ or π for $SL(3,\mathbb{Z})$, we determine an lower bound for the number of Hecke-Maass cusp forms Φ_j's in an orthonormal system such that Φ (or π) and Φ_j simultaneously acquire non-zero quadratically twisted central values. Our main result is derived from the calculation of moments of twisted central L-values, which is accomplished through the application of Waldspurger-type formulas for Maass cusp forms and the Kuznetsov trace formula for the Kohnen subspace.

Jake Huryn (OSU):

p-adic compatibility of canonical local systems on adjoint Shimura varieties

Given a number field F and an F-point y on a Shimura variety attached to a connected reductive group G, one can construct a collection of Galois representations

$$\{\rho_{y,\ell}\colon \mathrm{Gal}(\overline{\mathbb{Q}}/F) \to G(\mathbb{Q}_\ell)\}_\ell.$$

In this setting, Klevdal–Patrikis have recently established the following "ℓ-to-ℓ'" compatibility property: For any $\xi \in \mathrm{Rep}(G^{\mathrm{ad}})$ and all but finitely many places v of F, the characteristic polynomial of $(\xi \circ \rho_{y,\ell})(\mathrm{Frob}_v)$ has coefficients in $\mathbb{Q}$ and does not depend on ℓ when $v \nmid \ell$. Our work proves a similar "ℓ-to-p" compatibility which takes into account the situation when $v \mid \ell$ and involves the p-adic Hodge theory of $\xi \circ \rho_{y,\ell}$.

For Galois representations coming from geometry via étale cohomology, this "ℓ-to-p" compatibility follows from a theorem of Katz–Messing. Our results apply in particular to all Shimura varieties not of Abelian type (having rational weight cocharacter), the case in which a geometric origin of $\rho_{y,\ell}$ remains elusive.

Our arguments make key use of several very recent and very deep developments in p-adic geometry: results of Esnault–Groechenig on the p-adic properties of rigid connections, the algebraic p-adic Riemann–Hilbert functor of Diao–Lan–Liu–Zhu, and Kedlaya's proof of Deligne's "*petits camarades cristallins*" conjecture for smooth varieties.

This is joint work with Kiran Kedlaya, Christian Klevdal, and Stefan Patrikis.

Will Newman (OSU):

Injective Quadratic Self-Maps on $\mathbb{P}^2$

In this poster, I describe a criterion for when a quadratic rational map $\mathbb{P}^2 \dashrightarrow \mathbb{P}^2$ defined over a field K is injective on K-rational points, along the way giving some partial results for $\mathbb{P}^n$. When K is a finite field, the criterion is very restrictive, and we are able to explicitly describe exactly which self-maps of $\mathbb{P}^2$ are injective (hence, bijective) on K-rational points. This is joint work with Michael Zieve.

Peikai Qi (Michigan State University):

An analogue of Greenberg pseudo-null conjecture for CM fields

We will give an analogue of Greenberg's pseudo-null conjecture for CM fields. Let K be a CM field and K^+ be the unique totally real subfield of K. Assume that primes above p in K^+ all splits in K. Let $\mathfrak{P}_1, \mathfrak{P}_2, \cdots, \mathfrak{P}_s, \tilde{\mathfrak{P}}_1, \tilde{\mathfrak{P}}_2, \cdots, \tilde{\mathfrak{P}}_s$ be prime ideas in K above p, where $\tilde{\mathfrak{P}}_i$ is the complex conjugation of $\mathfrak{P}_i$. We show that there is unique $\mathbb{Z}_p$-extension of K unramified outside $\mathfrak{P}_1, \mathfrak{P}_2, \cdots, \mathfrak{P}_s$ if Leopoldt's conjecture holds for K. We also show that such $\mathbb{Z}_p$-extension for CM field has similar properties as cyclotomic $\mathbb{Z}_p$-extension of a totally real field. We also give some criteria for Iwasawa invariant $\mu = \lambda = 0$. The work is joint with Matt Stokes.

Neelarnab Raha (Penn State University):

Brill-Noether theory for vector bundles on surfaces

The Brill-Noether theory of curves plays a fundamental role in the theory of curves and their moduli and has been intensively studied since the 19th century. In contrast, Brill-Noether theory for higher dimensional varieties is less understood. It is hard to determine when Brill-Noether loci are nonempty and these loci can be reducible and of larger than the expected dimension.

Let E be a semistable vector bundle on $\mathbb{P}^2$. We give an upper bound $\beta_{r,\mu}$ for $h^0(E)$ in terms of the rank r and the slope μ of E. We show that the bound is achieved precisely when E is a Steiner bundle. We classify those E for which $h^0(E)$ is within $\lfloor \mu \rfloor + 1$ of $\beta_{r,\mu}$. We determine the nonemptiness, irreducibility and dimension of the Brill-Noether loci with $h^0(E)$ in this range. When they are proper subvarieties, these Brill-Noether loci are irreducible though almost always of larger than the expected dimension. This is joint work with Izzet Coskun and Jack Huizenga.

We also have a similar bound on $\mathbb{P}^1 \times \mathbb{P}^1$, although it is more subtle depending on how balanced the first Chern class is, and what global generation properties the vector bundle possesses.

Laurence Petrus Wijaya (University of Kentucky):

Sums of Kloosterman sums over square-free and smooth integers

Recently there has been a large number of works on bilinear sums with Kloosterman sums and on sums of Kloosterman sums twisted by arithmetic functions. Motivated by these, we consider several related new questions about sums of Kloosterman sums parameterised by square-free and smooth integers. This is joint work with Xuancheng Shao and Igor Shparlinski.

Shifan Zhao (OSU):

Low-lying zeros of Hilbert modular L-functions weighted by powers of central L-values

The famous Katz-Sarnak philosophy predicts that the distribution of low-lying zeros of families of L-functions follow certain symmetry through a Random Matrix Model. Recent studies in this field focus on the change of symmetry type once the distribution is weighted by central L-values. In this talk I will present recent results towards this problem for L-functions of Hilbert modular forms defined over totally real number fields, weighed by first, second, and third powers of central L-values, respectively. This is based on joint work with Zhining Wei and Liyang Yang.